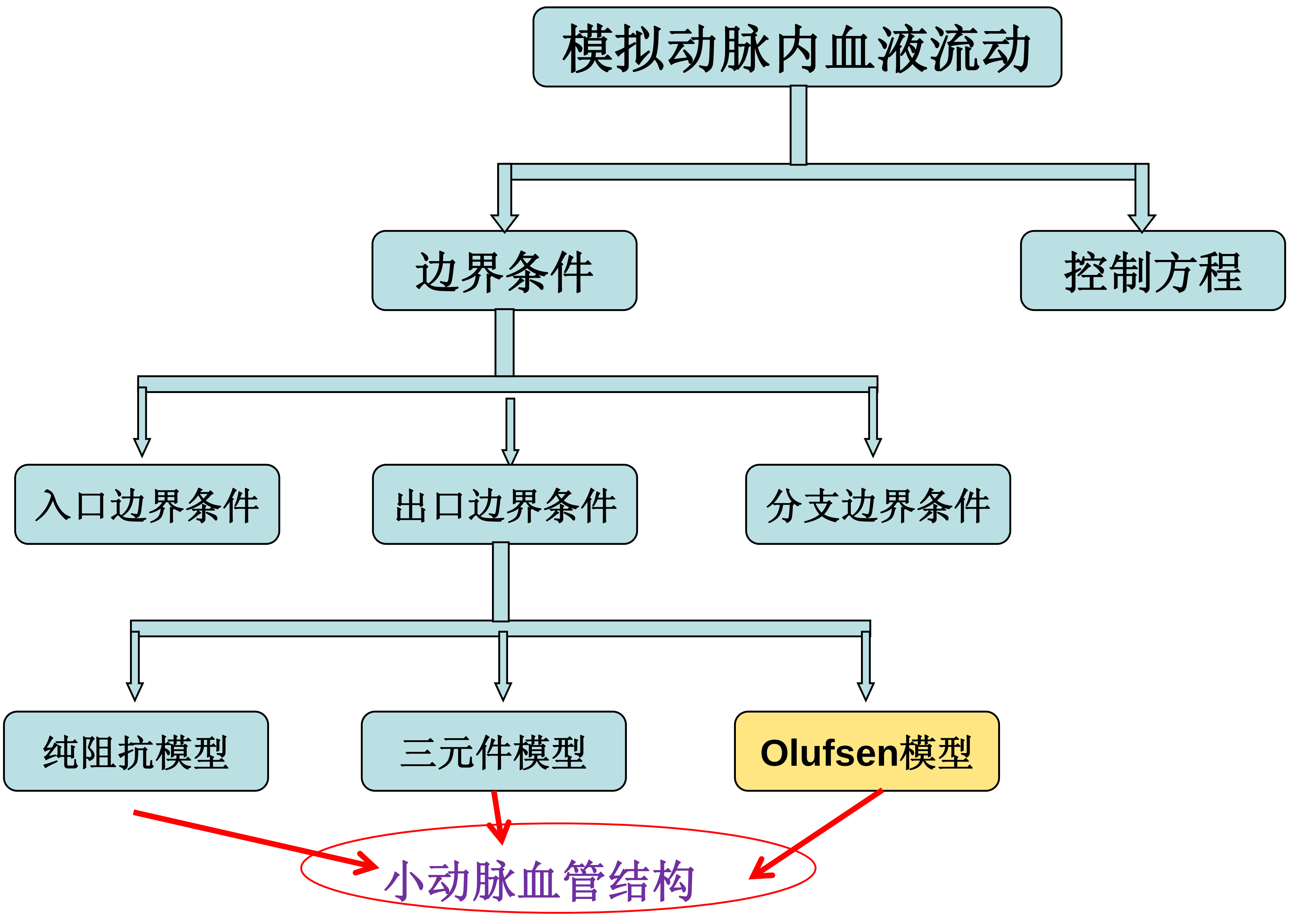
小血管网结构树模型
Fluid Dynamic Model in Structured Small Arteries

Olufsen, MS, Am J Physiol Heart Circ Physio 276:257-268, 1999

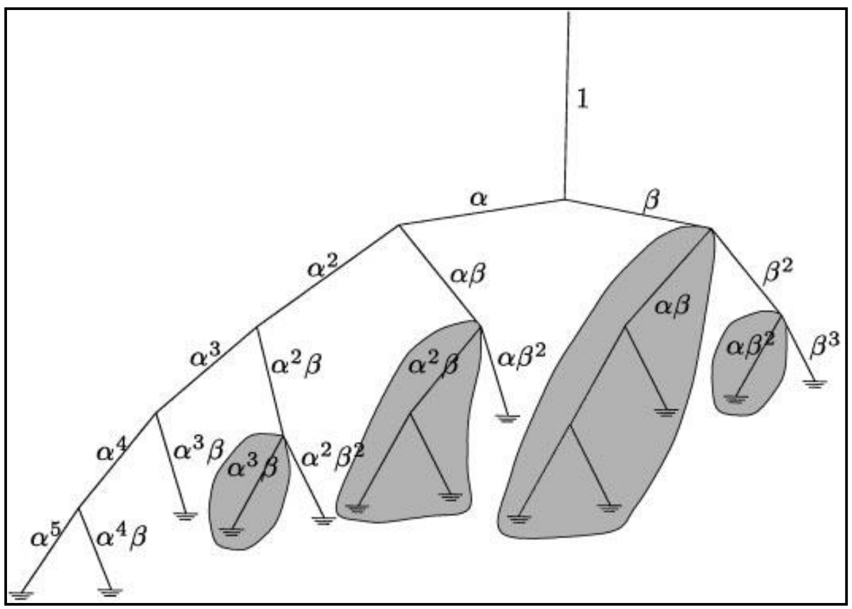
姬长金 2011力学大会CSTAM2011-E03-0152

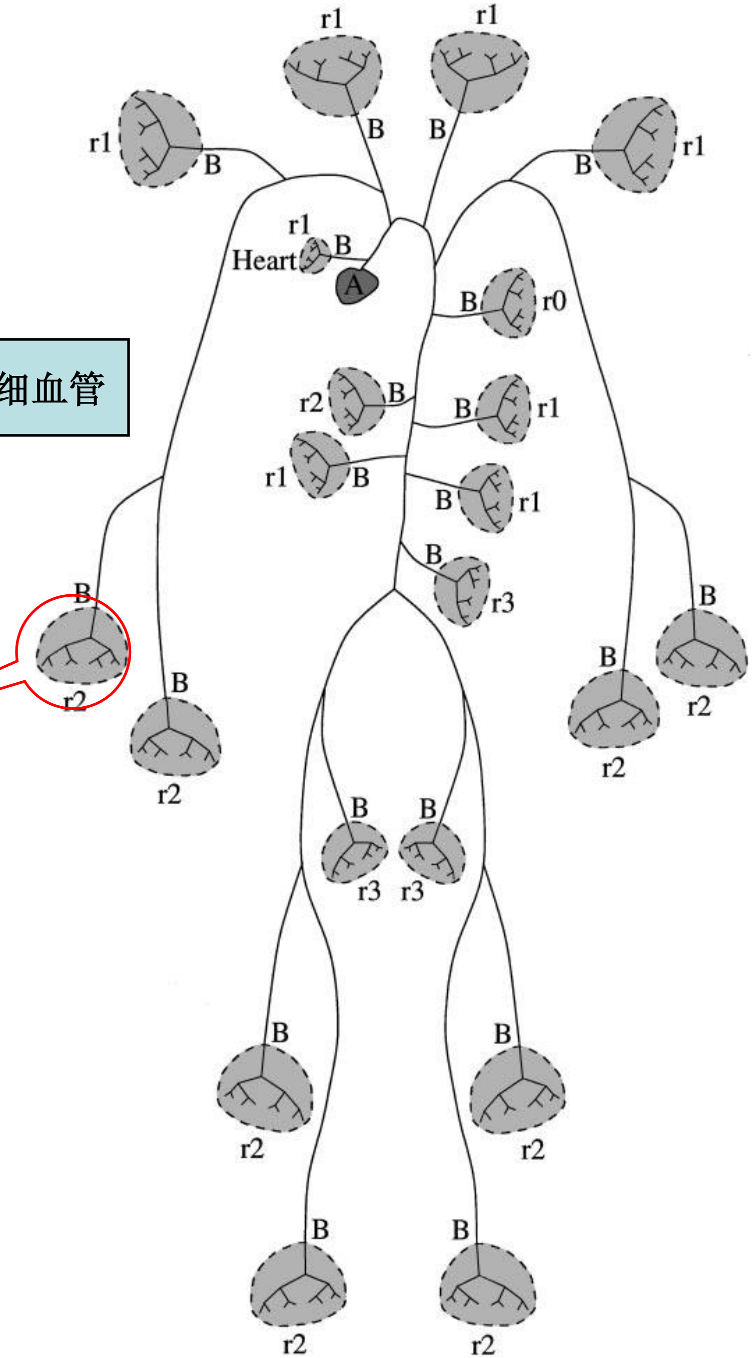


模拟动脉内血液流动
边界条件
控制方程
入口边界条件
出口边界条件
分支边界条件
纯阻抗模型
三元件模型
Olufsen模型
小动脉血管结构

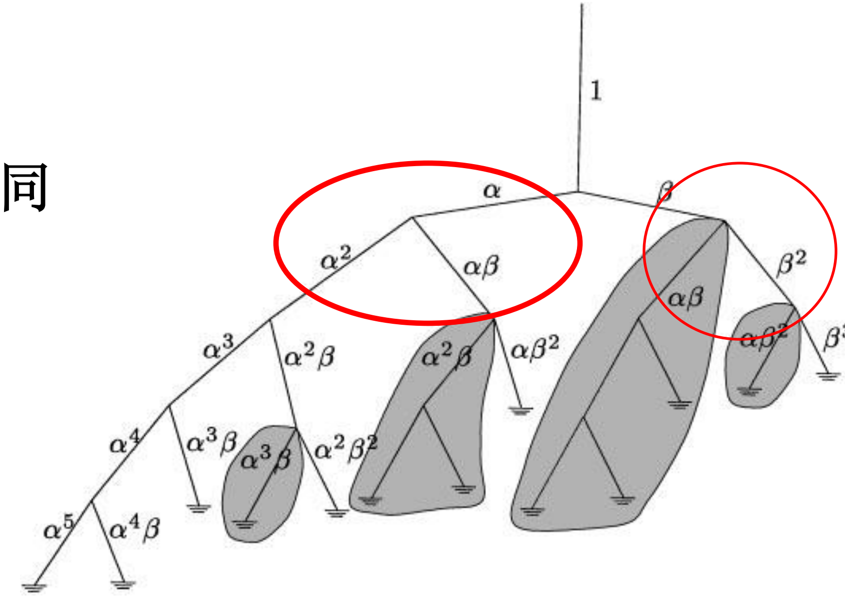
人为构建血管拓扑结构模拟大动脉截断处之后的真实血管结构

- 动脉 — 感兴趣血管，详细求解
- 小动脉 — 人为构建，使动脉的边界延伸到毛细血管
- 毛细血管 — 压力稳定，边界容易确定





如何构建血管树



假设各级血管向下分支的半径比例关系相同

$$r_{d1} = \alpha r_p$$

$$r_{d2} = \beta r_p$$

半径关系 (Uylings,1977;血管最小功耗原理)

$$r_p^{\ \varepsilon} = r_{d1}^{\ \varepsilon} + r_{d2}^{\ \varepsilon}$$

Laminar flow $\varepsilon = 3.0$

Turbulent flow $\varepsilon = 2.33$

非对称关系 (Zamir,1978)

Area-ratio $\eta = \dfrac{r_{d1}^{\ 2} + r_{d2}^{\ 2}}{r_p^{\ 2}}$

asymmetry-ratio $\gamma = \left(\dfrac{r_{d1}}{r_{d2}}\right)^2$

$$\alpha = (1+\gamma^{\varepsilon/2})^{-1/\varepsilon}$$

$$\beta = \alpha\sqrt{\gamma}$$

$\varepsilon = 2.76$

$\gamma = 0.41$

$\alpha = 0.9$

$\beta = 0.6$

给定最小半径值r_{min}，即可构建出末梢血管的结构血管树

小血管树模型

⇓

- 流体流动
- 壁面运动
- 流体和壁面作用

⟹ 线性化并进行求解

$$w_r = \frac{p_c}{w_0}\left(-\frac{\beta_0^2 a}{\mu w_0} + \frac{A}{J_0(w_0)} J_0\left(\frac{w_0 r}{a}\right)\right)$$

$$\beta_0 = \frac{ia\omega}{c} \quad w_0 = i^3 \frac{a^2\omega}{\nu} = i^3 w^2$$

$$Q = \int_0^a w_r 2\pi r dr$$

$$i\omega Q + \frac{A_0(1-F_J)}{\rho}\frac{\partial P}{\partial x} = 0$$

$$\frac{\partial A}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad +$$

$$p(x,t) = \sum_{k=-\infty}^{\infty} P(x,\omega_k)e^{i\omega_k t}$$

$$q(x,t) = \sum_{k=-\infty}^{\infty} Q(x,\omega_k)e^{i\omega_k t}$$

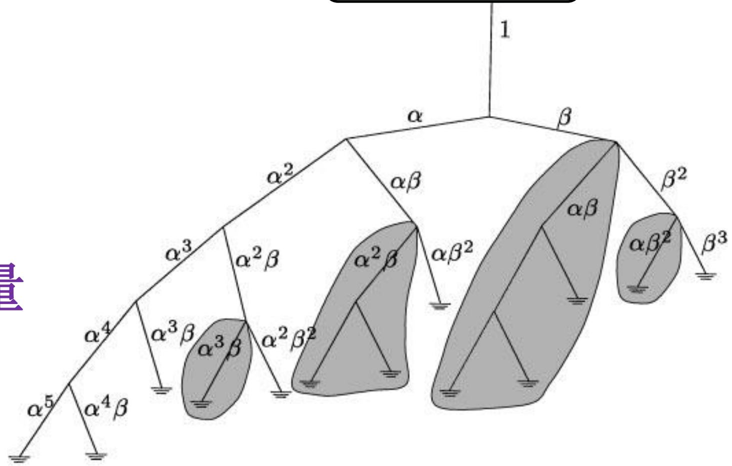
⟹

$$i\omega CP + \frac{\partial Q}{\partial x} = 0$$

将p和q用傅里叶级数展开

小血管树的模拟求解

Z(0,w)？



结构树的目的是为了求解出根血管的压力流量关系作为大血管的出口边界条件，如何求解？

小血管树根部阻抗

Impedance

对于每一个频率w_k,都满足

$$i\omega CP + \frac{\partial Q}{\partial x} = 0$$

$$i\omega Q + \frac{A_0(1-F_j)}{\rho}\frac{\partial P}{\partial x} = 0$$

！ ！

$$Z(x,\omega) = \frac{ig^{-1}(b\cos(\omega x/c)) - a\sin(\omega x/c)}{a\cos(\omega x/c) + b\sin(\omega x/c)}$$

？ ？

如果已知 $Z(L,\omega)$

$$Z(0,\omega) = \frac{ig^{-1}\sin(\omega L/c) - Z(L,\omega)\cos(\omega L/c)}{\cos(\omega L/c) + igZ(L,\omega)\sin(\omega L/c)}$$

$Z(L,\omega)$ b/a

$\omega \to 0$

$$Z(0,0) = \frac{8\mu l_{rr}}{\pi r_0^3} + Z(L,0) = \frac{8\mu l_{rr}}{\pi r_0^3}\frac{2\alpha^3 - (\frac{1}{2\alpha^3})^n}{2\alpha^3 - 1}$$

参数定义

Wave propagation speed C

$$c = \sqrt{\frac{A_0(1-F_j)}{\rho C}}$$

Scalar g

$$g = \sqrt{CA_0(1-F_j)/\rho}$$

Vessel Compliance

$$C = \frac{dA}{dp} = \frac{3A_0 r}{2Eh} \qquad (Eh >> pr)$$

$$F_J(w) = \begin{cases} 2/(wi^{1/2})[1+(2w)^{-1}+O(w^{-2})] & w \to \infty \\ 1-i(w^2/8)-(w^4/48)+O(w^6) & w \to 0 \end{cases}$$

Womersley number

$$w = r\sqrt{\omega/\upsilon}$$

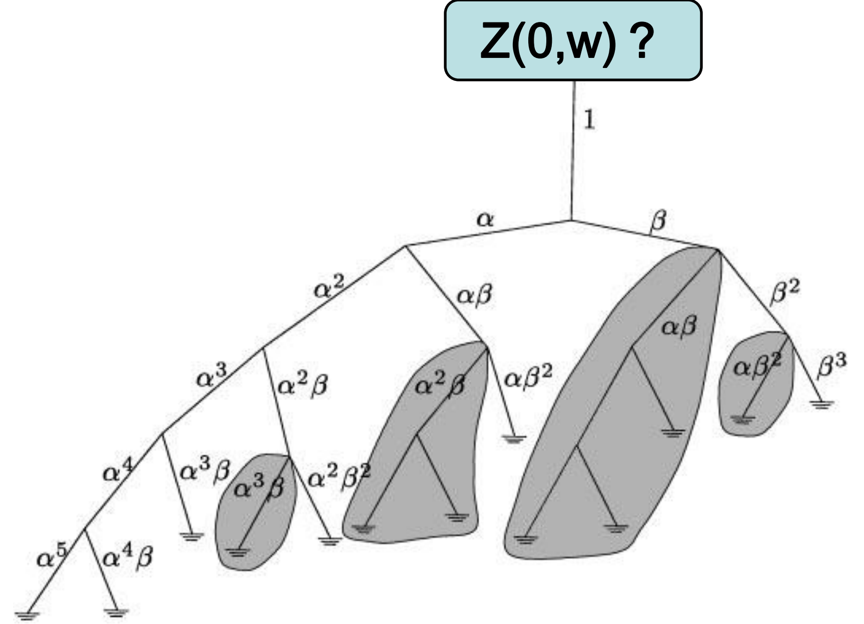
Elastic relation

$$\frac{Eh}{r_0} = k_1 e^{k_2} + k_3$$

各级阻抗传递?

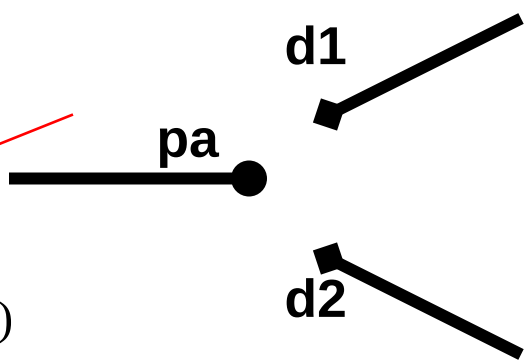
单根血管由末端传向始端

$$Z(0,\omega)=\frac{ig^{-1}\sin(\omega L/c)-Z(L,\omega)\cos(\omega L/c)}{\cos(\omega L/c)+igZ(L,\omega)\sin(\omega L/c)}$$

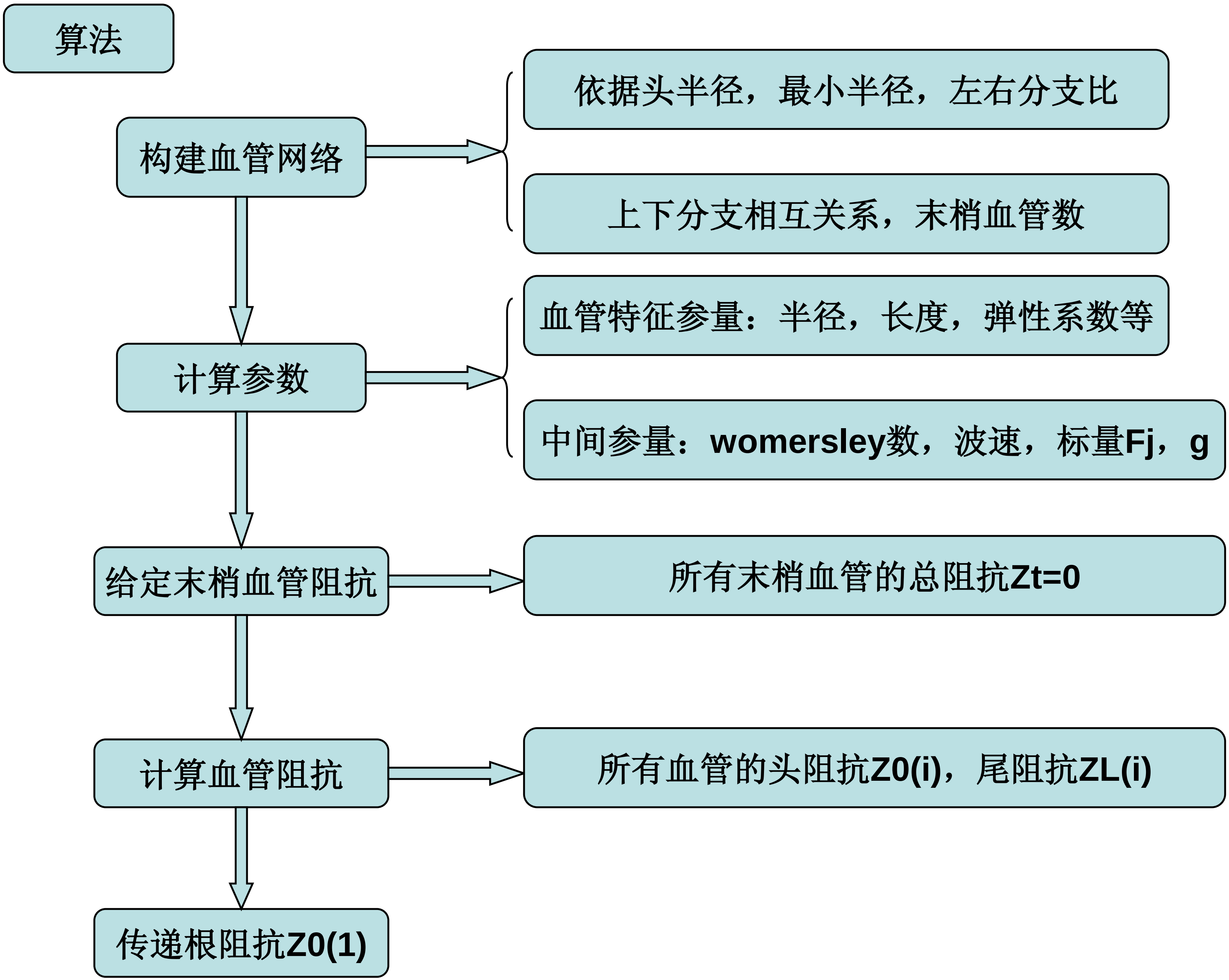


分支血管由子血管传向母血管

$$\frac{1}{Z_{pa}(L,\omega)}=\frac{1}{Z_{d1}(0,\omega)}+\frac{1}{Z_{d2}(0,\omega)}$$



给定终端阻抗Z_t ,就可计算出根血管 $Z(0,\omega)$


算法
构建血管网络
依据头半径，最小半径，左右分支比
上下分支相互关系，末梢血管数
计算参数
血管特征参量：半径，长度，弹性系数等
中间参量：womersley数，波速，标量Fj，g
给定末梢血管阻抗
所有末梢血管的总阻抗Zt=0
计算血管阻抗
所有血管的头阻抗Z0(i)，尾阻抗ZL(i)
传递根阻抗Z0(1)

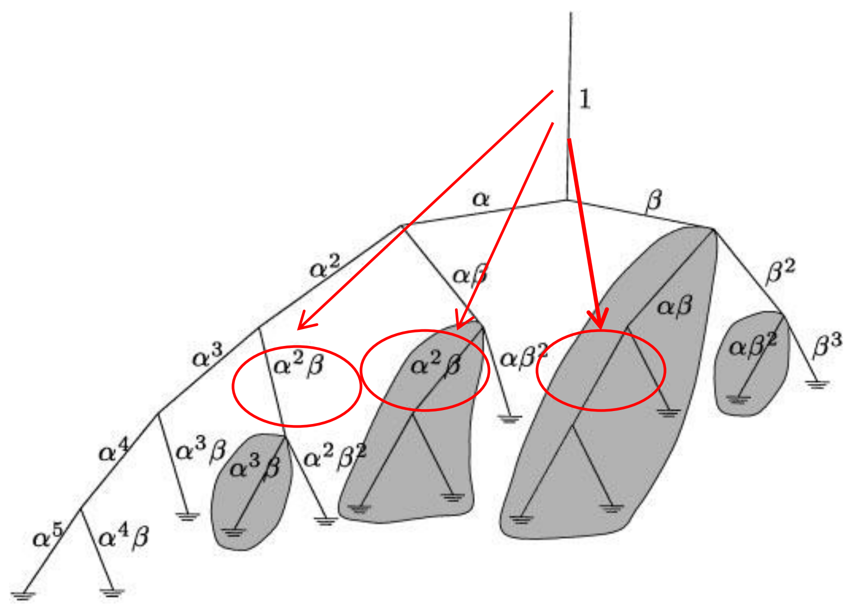
建立满二叉树：

有很大的局限性，当**R_root=2.5e-3,R_min=1.0e-4**时，二叉树的血管数有：**2^{32}-1**，数据量太大，超出了**fortran**整型变量的声明范围

使用递归算法节约存储空间

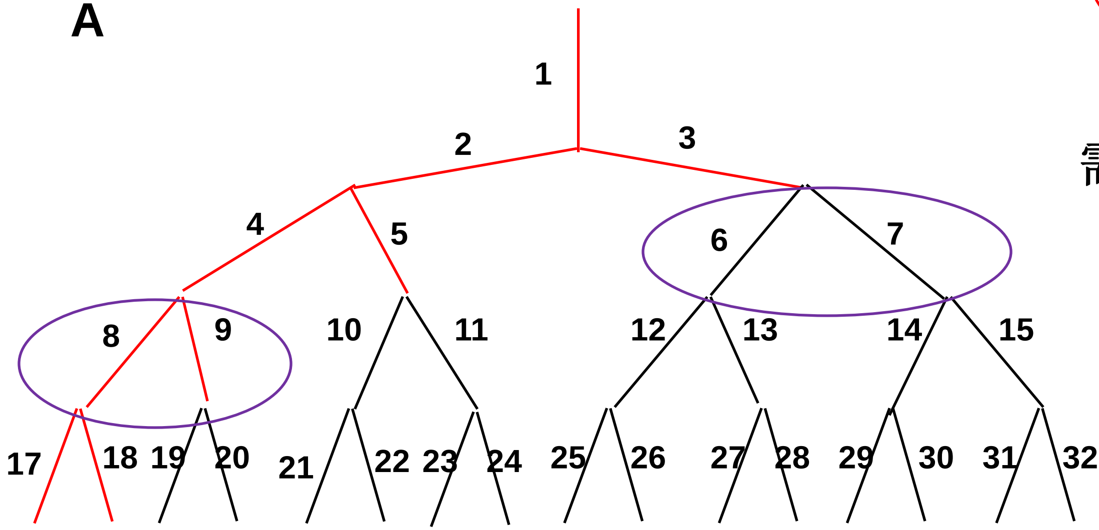
Recursive algorithm

只存储一根血管的数据

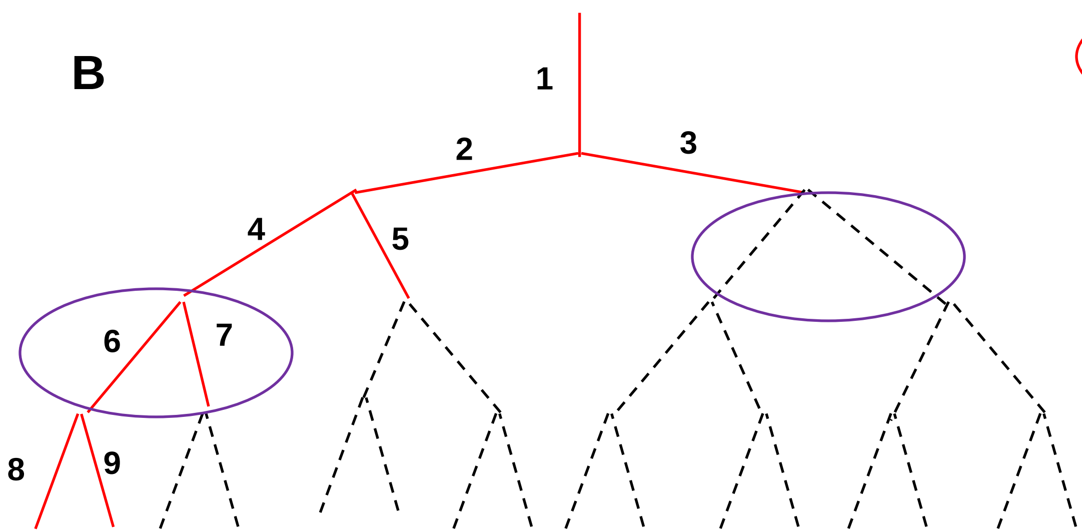


优化结构树程序，踢掉血管树中无效的血管

A



B



A(nnode) 存储满二叉树

B(nart) 存储有效血管数

需要知道 **A**与**B**之间的对应关系？

A=B+无效血管

算法：

del=0 ! 记录无效血管数

do i=1,nnode

if r(i)<r_min

del=del+1

end if

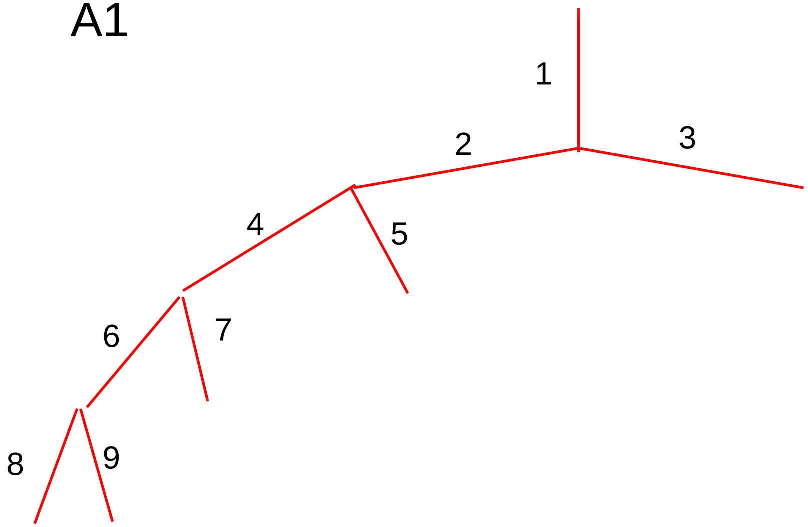
j=i-del

tree(j)=i

End do

A(i)与B(j)是一一对应的关系

A1



影响参数

Rroot

r_min

Alpha，beta

Irr

Zt_bot

调试分析

$$z(0,t)=\sum_{k=-n}^{n} Z(0,\omega_k)e^{i\omega_k t}$$

$$\omega_k=\frac{2\pi}{T}*k$$

n=10

$Z(0,\omega_k)$

```
(3.3662125E+08,-7.5594240E+08)
(2.9121757E+08,-6.5557350E+08)
(1.8197080E+08,-4.8311946E+08)
(7.5548200E+07,-2.6527648E+08)
(7.3396088E+07,-2.3663760E+08)
(7.2172928E+07,-2.0897616E+08)
(7.2534584E+07,-1.8190056E+08)
(5.8748852E+07,-1.1567906E+08)
(9589628.,-3.2911096E+07)
(2.9041030E+07,-5.3061588E+07)
(1.0687749E+08,0.0000000E+00)
(2.9041030E+07,5.3061588E+07)
(9589628.,3.2911096E+07)
(5.8748852E+07,1.1567906E+08)
(7.2534584E+07,1.8190056E+08)
(7.2172928E+07,2.0897616E+08)
(7.3396088E+07,2.3663760E+08)
(7.5548200E+07,2.6527648E+08)
(1.8197080E+08,4.8311946E+08)
(2.9121757E+08,6.5557350E+08)
(3.3662125E+08,7.5594240E+08)
Press any key to continue
```

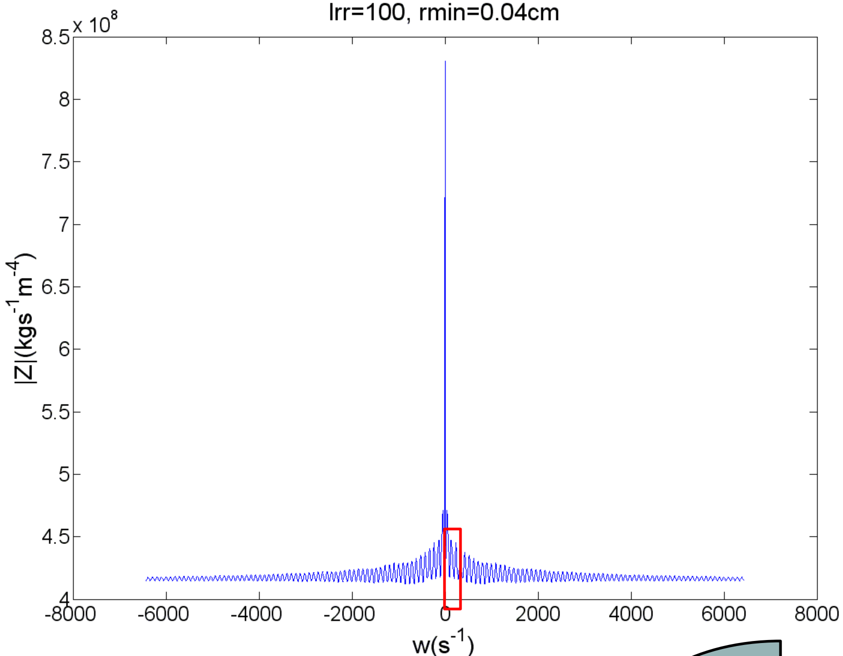
$|Z(0,\omega_k)|$

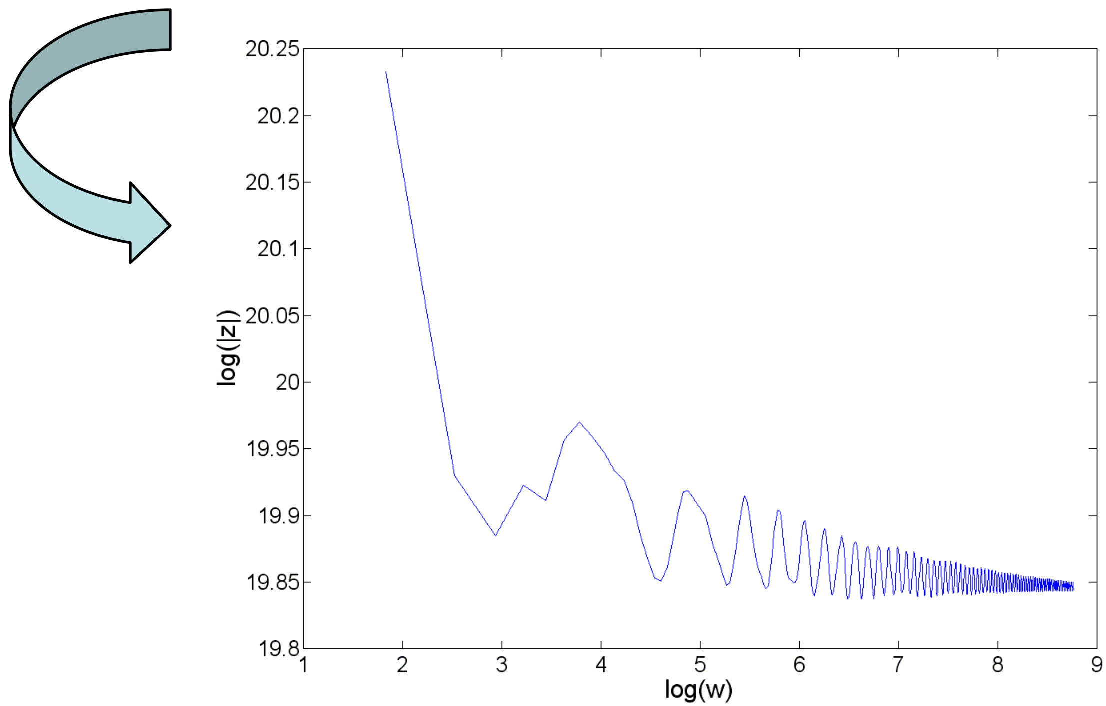
```
 8.2750394E+08
 7.1734528E+08
 5.1625360E+08
 2.7582448E+08
 2.4775862E+08
 2.2108814E+08
 1.9582922E+08
 1.2974233E+08
 3.4279748E+07
 6.0488952E+07
(1.0687749E+08,0.0000000E+00)
 6.0488952E+07
 3.4279748E+07
 1.2974233E+08
 1.9582922E+08
 2.2108814E+08
 2.4775862E+08
 2.7582448E+08
 5.1625360E+08
 7.1734528E+08
 8.2750394E+08
Press any key to continue
```

phase

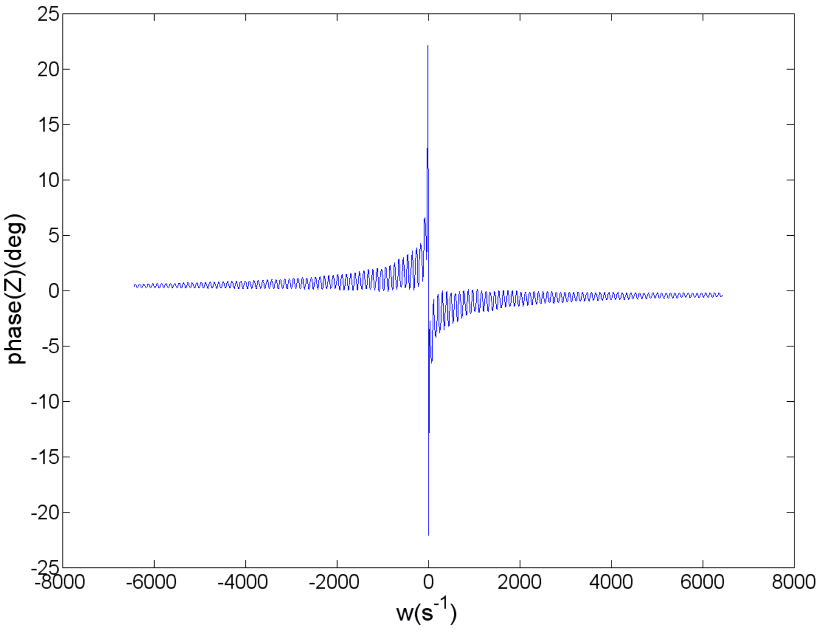
```
 -65.99658
 -66.04835
 -69.36072
 -74.10355
 -72.76807
 -70.94691
 -68.25989
 -63.07578
 -73.75494
 -61.30776
(1.0687749E+08,0.0000000E+00)
  61.30776
  73.75494
  63.07578
  68.25989
  70.94691
  72.76807
  74.10355
  69.36072
  66.04835
  65.99658
Press any key to continue
```

阻抗图

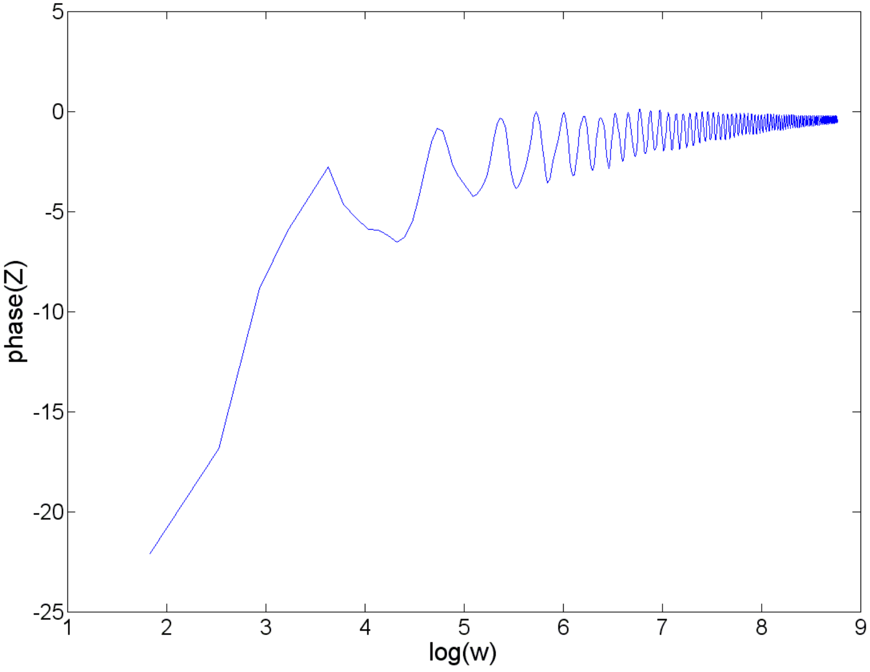




相位图



25
20
15
10
5
0
-5
-10
-15
-20
-25
phase(Z)(deg)
-8000
-6000
-4000
-2000
0
2000
4000
6000
8000
w(s-1)



5
0
-5
-10
-15
-20
-25
phase(Z)
1
2
3
4
5
6
7
8
9
log(w)

总结：

1. 根半径变化，根阻抗幅值的振荡频率变化很显著

2. 最小半径和zt_bot的作用比较相似，Z(0,0)受影响显著，其他频域，尤其是高频部分受影响较小

3. Alpha/beta的比值越接近1，影响|Z|的振荡幅度