Probability of Color Rearrangement at Partonic Level in Hadronic W^+W^- Decays

Zong-Guo Si[a], Qun Wang[a], Qu-Bing Xie[b,a]

a Department of Physics, Shandong University

Jinan Shandong 250100, P.R. China

b Center of Theoretical Physics, CCAST(World Lab),

Beijing 100080, P. R. China

A strict method to calculate the color rearrangement probability at partonic level in hadronic W^+W^- decays is proposed. The color effective Hamiltonian H_c is constructed from invariant amplitude for the process $e^+e^- \to W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4 + ng$ ($n = 0,\ 1,\ 2,\ \cdots$) and is used to calculate the cross sections of various color singlets and the color rearrangement probability. The true meaning of the color rearrangement is clarified and its difference from color interference is discussed.

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I. INTRODUCTION

At LEP2 energy, the real W^+W^- pair production through e^+e^- annihilation becomes possible. More accurate measurements on W mass (M_W) and other properties can be made at this stage of LEP project. The success of the precision measurements of M_W relies on accurate theoretical knowledge of the dynamics of the production and decay stages in $e^+e^- \to W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4$. However, the possible Color Rearrangement (CR) may obscure the separate identities of two W bosons so that the final hadronic state may no longer be considered as a superposition of two separate W decays. Thus the W mass determination may be distorted by the color reconnection effect in hadronic W-pair decays. This effect was first studied by Gustafson, Pettersson and Zerwas (GPZ) [1]. It attracts a lot of studies in recent years [2–5]. Considering that the two Ws decay into two quark pairs $q_1\overline{q}_2$ and $q_3\overline{q}_4$, GPZ assume that these two original color singlets can be rearranged into two new ones $q_1\overline{q}_4$ and $q_3\overline{q}_2$ with a probability $(\frac{1}{9})$, and then energetic gluons are emitted independently within each new singlet, which implies that the CR occurs before the parton shower process begins. But Sjöstrand and Khoze do not regard this instantaneous scenario as a very likely one [2,3]. The reason is that the decay vertices of two W bosons are in general separated in space-time, and therefore the hard gluons (with $E_g \geq \Gamma_W$) are produced incoherently by the two pairs $q_1\overline{q}_2$ and $q_3\overline{q}_4$ [2,6]. So there are two color singlets C_1 and C_2, each containing a $q\overline{q}$ pair and a set of gluons. Furthermore they argue that the CR in Perturbative QCD (PQCD) phase only comes from the color interference which should be very small. Hence they conclude that the non-perturbative contribution dominates the CR effect because the two color singlets C_1 and C_2 coexist later during the relatively larger space-time scale of hadronization compared with that of $W^{\pm}$'s life. The non-perturbative CR probability is controlled by the space-time overlaps of the color field induced by two groups of partons in C_1 and C_2. Later on, Gustafson and Häkkinen stress that CR can only originate from the partonic level and argue that the hard gluon emission will enlarge the CR effect. Because there are increasing ways of color recombination between the partons of C_1 and those of C_2 with the growing number of emitted gluons, even though the rearrangement probability is only $\frac{1}{N_c^2}$ for each way, the total probability at the partonic level may be greatly enhanced to the order $\sim (l+1)(m+1)/N_c^2$ where l and m are the numbers of gluons in C_1 and C_2 respectively. The final probability can in principle be modified by multiplying the factor of order $\sim (l+1)(m+1)/N_c^2$ by unknown functions of variables which characterize the space-time overlaps of the color fields induced by partons of C_1 and C_2. But Gustafson and Häkkinen's analysis on the total probability at the partonic level is only a qualitative one. It does not include many other ways of forming singlets and the calculation is not based on a strict formulation.

We should keep in mind that the color fields stretched between partons must be restricted in a preconfined state, i.e. a singlet. Thus in PQCD phase, CR means the transformation from a set of original singlets to that of new ones. Hence the CR probability from C_1 (containing $q_1\overline{q}_2$) and C_2(containing $q_3\overline{q}_4$) to new recombined ones where $q_1\overline{q}_4$ and $q_3\overline{q}_2$ belong to different singlets should and can be estimated by PQCD with more reason and accuracy. In this paper, we try to calculate the rearrangement probability from a strict systematic approach of PQCD. This approach is based on the color effective Hamiltonian [7] H_c which is built from the recursive form [8–10] of the invariant amplitude M for the process:

$$e^+e^- \to W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4 + ng, \qquad n = 0,\ 1,\ 2,\ \cdots. \tag{1}$$

The color effective Hamiltonian H_c is used to calculate the cross sections and the probabilities of various rearranged color singlets formed by final partons. The physical significance of our approach lies in that it includes all of effects caused by the different space-time intervals between the decay vertices of two W-bosons. The CR probability we obtain shows that the CR of PQCD stage is not negligible. This seems different from what Sjöstrand and Khoze conclude in ref. [2,3]. In our approach, the meaning of CR can be clearly defined and the difference from color interference can be easily elucidated.

The outline of this paper is as follows: in section II, we give the invariant amplitude M_n for the process (1) in the recursive form; then the color effective Hamiltonian H_c is abstracted from M_n in section III; thirdly, we use H_c to analyze the color singlet structure of the parton states $q_1\overline{q}_2q_3\overline{q}_4 + ng$ with $n = 0,\ 1,\ 2$ and give the rearrangement probabilities in section IV; finally, a summary is given in the last section.

II. CROSS SECTION

The differential cross section $d\sigma_n$ for the process (1) is

$$d\sigma_n = \Phi|M_n|^2 d\wp_{n+4}(Q_1, Q_2, Q_3, Q_4, K_1, \cdots, K_n), \tag{2}$$

where $\Phi = \frac{1}{8s}$ is the flux factor multiplied by a spin average factor, and the phase space factor $d\wp_{n+4}$ is defined by

$$d\wp_{n+4} = (2\pi)^4\delta^4(P_+ + P_- - \sum_{i=1}^{4} Q_i - \sum_{j=1}^{n} K_j) \prod_{i=1}^{4} \frac{d^3\, \vec{Q}_i}{(2\pi)^3 2Q_i^0} \prod_{j=1}^{n} \frac{d^3\, \vec{K}_j}{(2\pi)^3 2K_j^0},$$

here P_+, P_-, Q_i ($i = 1,\ \cdots,\ 4$) and K_j ($j = 1,\ \cdots,\ n$) denote the 4-momenta of e^+, e^-, quarks and gluons respectively.

According to Feynman rules, the matrix element M_n of the process (1) (see fig.1) can be written as

$$\begin{aligned} M_n = & \sum_{m=0}^{n} \sum_{V=\nu_e,\gamma^*,Z^0} \pounds_{\nu\mu}^{V} D^{\nu\nu'}(W^{+2}) D^{\mu\mu'}(W^{-2}) \\ & \times \hat{S}_{\nu'}(Q_1; K_1, \cdots, K_m; Q_2) \hat{S}_{\mu'}(Q_3; K_{m+1}, \cdots, K_n; Q_4), \end{aligned} \tag{3}$$

where $W^{\pm}$ denotes the 4-momenta of $W^{\pm}$ boson, and $D^{\nu\mu}(W^{\pm 2})$ is the propagator of $W^{\pm}$ boson; $\pounds_{\nu\mu}^{V}$ is the polarization tensor of leptons; $\hat{S}_\nu$ is the current containing quarks and gluons which depends on the 4-momenta, helicities and color indices of the outgoing partons. In the recursive form, the current $\hat{S}_\nu$ can be expressed by

$$\hat{S}_\nu(Q_1; K_1, K_2, \cdots, K_m; Q_2) = ieg_s^m \sum_{P(1,2,\cdots,m)} (T^{a_1} T^{a_2} \cdots T^{a_m})_j^i S_\nu(Q_1; 1, 2, \cdots, m; Q_2), \tag{4}$$

where g_s is QCD coupling constant; $T^a = \frac{\lambda^a}{2}$ and λ^a is Gell-Mann matrix for $SU_c(3)$, and S_ν is the spinor current (for detail, see refs. [8,9]).

Substituting eq. (4) into eq. (3), we obtain

$$M_n = \sum_{m=0}^{n} \sum_{P(1,\cdots,m)} \sum_{P(m+1,\cdots,n)} (T^{a_1} \cdots T^{a_m})_j^i (T^{a_{m+1}} \cdots T^{a_n})_l^k X_{(q_1 g_1 \cdots g_m \overline{q}_2)(q_3 g_{m+1} \cdots g_n \overline{q}_4)}, \tag{5}$$

where the indices $i,\ k$ ($j,\ l$) denote the color (anticolor) of outgoing quark (antiquark), and

$$\begin{aligned} X_{(q_1 g_1 \cdots g_m \overline{q}_2)(q_3 g_{m+1} \cdots g_n \overline{q}_4)} = & -e^2 g_s^n \sum_{V=\nu_e,\gamma^*,Z^0} \pounds_{\nu\mu}^{V} D^{\nu\nu'}(W^{+2}) D^{\mu\mu'}(W^{-2}) \\ & \times S_{\nu'}(Q_1; K_1, \cdots, K_m; Q_2) S_{\mu'}(Q_3; K_{m+1}, \cdots, K_n; Q_4). \end{aligned}$$

$\pounds_{\nu\mu}^{V}$ can be written as follows

$$\begin{cases} \pounds_{\nu\mu}^{V} = \overline{v}(P_+)[ie\Gamma_\nu^W] \frac{iQ_\alpha \gamma^\alpha}{Q^2} [ie\Gamma_\mu^W] u(P_-), & V = \nu_e, \\ \pounds_{\nu\mu}^{V} = \overline{v}(P_+)[ie\Gamma_\alpha^V] u(P_-) \tilde{D}^{(V)\alpha\beta}(Q'^2)[ieF_{\beta\nu\mu}^{V}(Q', W^+, W^-)], & V = \gamma^*, Z^0, \end{cases} \tag{6}$$

where the repetition of indices represents summing (we use this convention unless explicitly specified); Γ^B_α is the fermion-boson vertices for the boson $B = \gamma^*,\ Z^0,\ W^\pm$; $Q = W^+ - P_+$ and $Q' = P_+ + P_-$; $\tilde{D}^{V\alpha\beta}(Q'^2)$ is the propagator of the vector boson V (=γ^* or Z^0); $F^V_{\beta\nu\mu}(Q', W^+, W^-)$ is the three-boson vertex of $\gamma^* W^+ W^-$ ($V = \gamma^*$) or $Z^0 W^+ W^-$ ($V = Z^0$).

Up to now, we use the recursive formula to write down the invariant amplitude M_n for the process $e^+e^- \to W^+W^- \to q_1\overline{q}_2 q_3\overline{q}_4 + ng$. In this form, we see that M_n can be clearly expressed as a uniform color part multiplied by a momentum function of partons. The effective Hamiltonian H_c can be found from this form of the amplitude. This is what we shall do in the next section.

III. COLOR EFFECTIVE HAMILTONIAN H_C

In ref. [7], from PQCD, a strict formulation has been proposed to calculate the cross section of color singlets for the process $e^+e^- \to \gamma^*/Z^0 \to q\overline{q} + ng$. Now we use the same approach to abstract the color effective Hamiltonian H_c for the process (1) from the invariant amplitude M_n given in eq. (5). Then H_c is found in the following form:

$$\begin{aligned} H_c &= \sum_{m=0}^{n} \sum_{P(1,\cdots,m)} \sum_{P(m+1,\cdots,n)} (T^{a_1}\cdots T^{a_m})^i_j (T^{a_{m+1}}\cdots T^{a_n})^k_l X_{(q_1 g_1\cdots g_m \overline{q}_2)(q_3 g_{m+1}\cdots g_n \overline{q}_4)} \\ &\quad \times \Psi^+_{1i}\Psi^{j+}_2 \Psi^+_{3k}\Psi^{l+}_4 A^{a_1+}\cdots A^{a_n+} \\ &= (\tfrac{1}{\sqrt{2}})^n \sum_{m=0}^{n} \sum_{P(1,\cdots,m)} \sum_{P(m+1,\cdots,n)} [\Psi^+_{1i}\Psi^{j+}_2 (G_1\cdots G_m)^i_j][\Psi^+_{3k}\Psi^{l+}_4 (G_{m+1}\cdots G_n)^k_l] \\ &\quad \times X_{(q_1 g_1\cdots g_m \overline{q}_2)(q_3 g_{m+1}\cdots g_n \overline{q}_4)}, \end{aligned} \tag{7}$$

where $\Psi^+_{ui} = (R^+, Y^+, B^+)_u$ and $\Psi^{j+}_u = (\overline{R}^+, \overline{Y}^+, \overline{B}^+)_u$ are the color creation operator for quark and antiquark, respectively. The color octet operator G_u of gluon u is defined by

$$G^i_{uj} = \frac{1}{\sqrt{2}}(\lambda^{a_u} A^{a_u}_u)^i_j = \Psi^{i+}_u \Psi^+_{uj} - \frac{1}{3}\Psi^{x+}_u \Psi^+_{ux}\delta^i_j, \tag{8}$$

where $A^{a_u}_u$ ($a_u = 1, \cdots, 8$) is color operator for gluon u and is defined in ref. [7]. Here H_c is another expression of S matrix, so it is not necessarily Hermitian.

For a final color state $|f>$, its cross section can be calculated by

$$\sigma_n^f = \int \Phi | < f|H_c|0 > |^2 d\wp_{n+4}, \tag{9}$$

where $|0>$ is the initial state free of color. If

$$|f >= |[\Psi_{1i}^+ \Psi_2^{j+} (G_1 \cdots G_m)_j^i][\Psi_{3k}^+ \Psi_4^{l+} (G_{m+1} \cdots G_n)_l^k] >,$$

after summing over all of the color indices, we have

$$\begin{aligned} \sum_f \sigma_n^f &= \int \Phi \sum_f | < f|H_c|0 > |^2 d\wp_{n+4} = \int \Phi < 0|H_c^+ H_c|0 > d\wp_{n+4} \\ &= \int \Phi |M_n|^2 d\wp_{n+4} = \sigma_n. \end{aligned} \tag{10}$$

It shows that the calculation of the ordinary cross section via H_c returns to the original form. So the validity of H_c for the process (1) is verified.

For the parton system $q_1\overline{q}_2 q_3 \overline{q}_4 + ng$, the color state is composed of the color charges of q_1, $\overline{q}_2$, q_3, $\overline{q}_4$ and n gluons. It belongs to the color space

$$3_1 \bigotimes 3_2^* \bigotimes 3_3 \bigotimes 3_4^* \bigotimes 8_1 \bigotimes \cdots \bigotimes 8_n.$$

There are many ways of reducing this color space. Corresponding to each reducible way is one set of orthogonal singlet sub-spaces whose bases contribute a complete and orthogonal set of color singlets. For a color singlet set $\{|f_k >, \ k = 1, \ \cdots\}$, we have

$$|f_k >< f_k| = 1, \qquad < f_k|f_l >= \delta_{kl} \tag{11}$$

and

$$\begin{aligned} \sum_k \sigma_n^k &= \int \Phi \sum_k | < f_k|H_c|0 > |^2 d\wp_{n+4} = \int \Phi < 0|H_c^+|f_k >< f_k|H_c|0 > d\wp_{n+4} \\ &= \int \Phi < 0|H_c^+ H_c|0 > d\wp_{n+4} = \int \Phi |M_n|^2 d\wp_{n+4} = \sigma_n. \end{aligned} \tag{12}$$

This is the result of unitarity.

According to eq. (7), for instance, one can get the concrete expressions of color effective Hamiltonian H_{cn} for the the process (1) with $n = 0, \ 1, \ 2$ as follows:

a.) for the process $e^+e^- \to W^+W^- \to q_1\overline{q}_2 q_3 \overline{q}_4$ $(n = 0)$,

$$H_{c0} = (\Psi_{1i}^+ \Psi_2^{i+})(\Psi_{3j}^+ \Psi_4^{j+}) X_{(q_1\overline{q}_2)(q_3\overline{q}_4)}; \tag{13}$$

b.) for the process $e^+e^- \to W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4g_1$ $(n=1)$,

$$H_{c1} = H_{c1}^1 + H_{c1}^2, \tag{14}$$

where

$$\begin{aligned} H_{c1}^1 &= \tfrac{1}{\sqrt{2}}(\Psi_{1i}^+ G_{1j}^i \Psi_2^{j+})(\Psi_{3k}^+ \Psi_4^{k+}) X_{(q_1 g\overline{q}_2)(q_3\overline{q}_4)}, \\ H_{c1}^2 &= \tfrac{1}{\sqrt{2}}(\Psi_{1i}^+ \Psi_2^{i+})(\Psi_{3j}^+ G_{1k}^j \Psi_4^{k+}) X_{(q_1\overline{q}_2)(q_3 g\overline{q}_4)}; \end{aligned}$$

c.) for the process $e^+e^- \to W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4g_1g_2$ $(n=2)$,

$$H_{c2} = H_{c2}^1 + H_{c2}^2 + H_{c2}^3, \tag{15}$$

where

$$\begin{aligned} H_{c2}^1 &= \tfrac{1}{2}\sum_{P(1,2)}(\Psi_{1i}^+ G_{1j}^i G_{2k}^j \Psi_2^{k+})(\Psi_{3l}^+ \Psi_4^{l+}) X_{(q_1 g_1 g_2\overline{q}_2)(q_3\overline{q}_4)}, \\ H_{c2}^2 &= \tfrac{1}{2}\sum_{P(1,2)}(\Psi_{1i}^+ \Psi_2^{i+})(\Psi_{3j}^+ G_{1k}^j G_{2l}^k \Psi_4^{l+}) X_{(q_1\overline{q}_2)(q_3 g_1 g_2\overline{q}_4)}, \\ H_{c2}^3 &= \tfrac{1}{2}\sum_{P(1,2)}(\Psi_{1i}^+ G_{1j}^i \Psi_2^{j+})(\Psi_{3k}^+ G_{2l}^k \Psi_4^{l+}) X_{(q_1 g_1\overline{q}_2)(q_3 g_2\overline{q}_4)}. \end{aligned}$$

IV. COLOR SINGLET STRUCTURE OF THE FINAL PARTON SYSTEM

In this section, we try to use H_c derived in last section to study the color singlet structure of the final parton system, give the CR probability and compare our results with those of other authors. As the gluon number grows, it becomes more and more difficult to calculate σ_n, and the number of different ways of forming color singlets increase drastically. To illustrate what our approach is and how it works, we study only three lowest order cases: $q_1\overline{q}_2q_3\overline{q}_4$, $q_1\overline{q}_2q_3\overline{q}_4g_1$ and $q_1\overline{q}_2q_3\overline{q}_4g_1g_2$.

A.) For the parton system $q_1\overline{q}_2q_3\overline{q}_4$, due to the reasons given in ref. [3], we have no need to consider the color configurations resulting from the reductions of $3 \bigotimes 3 = 3^* \bigoplus 6$ and $3^* \bigotimes 3^* = 3 \bigoplus 6^*$. Hence the color space $3_1 \bigotimes 3_2^* \bigotimes 3_3 \bigotimes 3_4^*$ has only two reducible ways which correspond to the color configurations with and without CR. In the following, we will discuss these two cases in detail.

For the reducible way

$$(3_1 \bigotimes 3_2^*) \bigotimes (3_3 \bigotimes 3_4^*) \to (1_{12} \bigoplus 8_{12}) \bigotimes (1_{34} \bigoplus 8_{34}) \to (1_{12} \bigotimes 1_{34}) \bigoplus (8_{12} \bigotimes 8_{34}),$$

the color singlet set is $\{|\tilde{f}_0^i >, \ i = 1, \ 2\}$ where

$$|\tilde{f}_0^1 >= \frac{1}{3}|(\Psi_{1i}^+ \Psi_2^{i+})(\Psi_{3j}^+ \Psi_4^{j+}) >, \qquad |\tilde{f}_0^2 >= \frac{1}{\sqrt{8}}|Tr(G_{12}G_{34}) >, \tag{16}$$

where $G_{xyi}^k = \Psi_{xi}^+ \Psi_y^{k+} - \frac{1}{3}\Psi_{xl}^+ \Psi_y^{l+} \delta_i^k$ ($xy = 12$, or 34) denotes the color octet state formed by q_x and $\overline{q}_y$. In $|\tilde{f}_0^1 >$, $\Psi_{1i}^+ \Psi_2^{i+}$ and $\Psi_{3j}^+ \Psi_4^{j+}$ represent the two initial color singlets within $q_1\overline{q}_2$ and $q_3\overline{q}_4$. Defining the probabilities of $|\tilde{f}_0^i >$ $(i = 1, \ 2)$ as $\tilde{P}_0^i = \frac{\int \Phi |<\tilde{f}_0^i|H_{c0}|0>|^2 d\wp_4}{\sigma_0}$ where σ_0 is the cross section for $e^+e^- \to W^+W^- \to q_1\overline{q}_2 q_3\overline{q}_4$ and H_{c0} is given in eq. (13), we find that $\tilde{P}_0^1 = 100\%$ and $\tilde{P}_0^2 = 0$. This set corresponds to the original color configurations.

For the other reducible way

$$(3_1 \bigotimes 3_4^*) \bigotimes (3_3 \bigotimes 3_2^*) \to (1_{14} \bigoplus 8_{14}) \bigotimes (1_{32} \bigoplus 8_{32}) \to (1_{14} \bigotimes 1_{32}) \bigoplus (8_{14} \bigotimes 8_{32}),$$

the color singlet set is $\{|f_0^i >, \ i = 1, \ 2\}$ where

$$|f_0^1 >= \frac{1}{3}|(\Psi_{1i}^+ \Psi_4^{i+})(\Psi_{3j}^+ \Psi_2^{j+}) >, \qquad |f_0^2 >= \frac{1}{\sqrt{8}}|Tr(G_{14}G_{32}) > . \tag{17}$$

Their probabilities are given by

$$P_0^1 = \frac{\int \Phi| < f_0^1|H_{c0}|0 > |^2 d\wp_4}{\sigma_0} = \frac{1}{9}, \qquad P_0^2 = \frac{\int \Phi| < f_0^2|H_{c0}|0 > |^2 d\wp_4}{\sigma_0} = \frac{8}{9}. \tag{18}$$

One sees that the probability of $|f_0^1 >$, where $q_1\overline{q}_4$ and $q_3\overline{q}_2$ form two singlets $\Psi_{1i}^+ \Psi_4^{i+}$ and $\Psi_{3j}^+ \Psi_2^{j+}$, is $\frac{1}{9}$. This state is just the CR case first discussed by GPZ [1]. But here it clearly shows that the states with and without CR belong to two different completeness sets. In the new set, the state f_0^2 is the color singlet made up of two color octets which are formed from color charges of the pair $q_1\overline{q}_4$ and $q_3\overline{q}_2$ respectively. So all of the four partons must in principle hadronize as a unity in this state, though we do not know how to treat this kind of hadronization rigorously. Note that the state $|f_0^2 >$ is different from $|\tilde{f}_0^1 >$. But in LUND string fragmentation picture, in $|f_0^2 >$ which is built up by two octets, the neutral color flow connects q_1 with $\overline{q}_2$ and q_3 with $\overline{q}_4$, and the substrings stretched within the pair $q_1\overline{q}_2$ and $q_3\overline{q}_4$ are treated as subsinglets (in fact, at the partonic level, they are not singlets, but only color neutral objects). Then the hadronization result of $|f_0^2>$ certainly has no difference with those of $|\tilde{f}_0^1>$ in LUND model.

B.) The color space $3_1 \bigotimes 3_2^* \bigotimes 3_3 \bigotimes 3_4^* \bigotimes 8_1$ of the parton system $q_1\overline{q}_2q_3\overline{q}_4g_1$ can be reduced in two ways: one corresponds to the case with no CR, and the other to that with CR. Here we discuss the latter case. Corresponding to the following reducible way

$$
\begin{aligned}
&(3_1 \bigotimes 3_4^*) \bigotimes (3_3 \bigotimes 3_2^*) \bigotimes 8_1 \rightarrow (1_{14} \bigoplus 8_{14}) \bigotimes (1_{32} \bigoplus 8_{32}) \bigotimes 8_1 \\
\rightarrow\ &(1_{14} \bigotimes 8_{32} \bigotimes 8_1) \bigoplus (8_{14} \bigotimes 1_{32} \bigotimes 8_1) \bigoplus (8_{14} \bigotimes 8_{32} \bigotimes 8_1),
\end{aligned}
$$

the completeness set of color singlets is $\{|f_1^j>,\ j=1,\ \cdots,\ 4\}$ where

$$
\begin{cases}
|f_1^1> \ = \frac{1}{\sqrt{24}}|Tr(G_{14}G_1)(\Psi_{3x}^+\Psi_2^{x+})>,\ |f_1^2> \ = \frac{1}{\sqrt{24}}|Tr(G_{32}G_1)(\Psi_{1x}^+\Psi_4^{x+})>, \\
|f_1^3> \ = \sqrt{\frac{3}{80}}|Tr(\{G_{14},G_1\}G_{32})>, \quad |f_1^4> \ = \frac{1}{\sqrt{48}}|Tr([G_{14},G_1]G_{32})>,
\end{cases}
\tag{19}
$$

with

$$
\begin{cases}
\{G_1,G_2\}_k^i = G_{1l}^iG_{2k}^l + G_{2l}^iG_{1k}^l - \frac{2}{3}Tr(G_1G_2)\delta_k^i, \\
[G_1,G_2]_k^i = G_{1l}^iG_{2k}^l - G_{2l}^iG_{1k}^l.
\end{cases}
$$

Their probabilities P_1^i $(i=1,\ \cdots,\ 4)$ are given by

$$
\begin{aligned}
P_1^i \ &= \frac{1}{\sigma_1}\int \Phi| < f_1^i|H_{c1}|0> |^2 d\wp_5 = \frac{1}{\sigma_1}\int \Phi| < f_1^i|H_{c1}^1|0> + < f_1^i|H_{c1}^2|0> |^2 d\wp_5 \\
&= \frac{1}{\sigma_1}\int \Phi[\sum_{j=1}^{2} | < f_1^i|H_{c1}^j|0> |^2 + 2Re(<0|H_{c1}^{1+}|f_1^i>< f_1^i|H_{c1}^2|0>)]d\wp_5,
\end{aligned}
\tag{20}
$$

where σ_1 is the cross section of the process $e^+e^- \rightarrow W^+W^- \rightarrow q_1\overline{q}_2q_3\overline{q}_4g_1$, and H_{c1} is given in eq. (14). They are related to the energy $\sqrt{S}$ and PQCD parameters (e.g. α_s and Y_{min} etc.). $\int \Phi \sum_{j=1}^{2} | < f_1^i|H_{c1}^j|0> |^2 d\wp_5$ are proportional to σ_1. Thus in the last line of eq. (20), the first term does not depend on these quantities. The second term is the color rearrangement caused by the color interference. Even if it vanishes, the color rearrangement still exists. This shows that the color interference contributes to only part of CR. Additionally, it is easy to verify $\sum_{i=1}^{4} P_1^i = 1$ which is the natural result of eq. (12).

We notice that in $|f_1^1>$, $Tr(G_{14}G_1)$ is the color singlet formed by q_1, $\overline{q}_4$ and g_1, while $\Psi_{3x}^+\Psi_2^{x+}$ the color singlet by q_3 and $\overline{q}_2$. The case of $|f_1^2>$ is similar to that of f_1^1. These two states are just the CR states discussed by Gustafson and Häkkinen in their naive model [4]. But the meaning of the probabilities of these two states in their work is different from that in ours. For the process $e^+e^- \to W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4g_1$, they consider two possible cases: in one, g_1 is radiated from $q_1\overline{q}_2$ with a probability $\tilde{P}$, and in the other, g_1 is radiated from $q_3\overline{q}_4$ with a probability $(1-\tilde{P})$. There are no interference terms. For each case, they give the CR probability $\frac{2}{9}$. So according to their analysis, one can derive that the total CR probability is $\frac{2}{9}$ for the whole process, since $\frac{2}{9}\tilde{P} + \frac{2}{9}(1-\tilde{P}) = \frac{2}{9}$. But in our calculation, the probability is expressed in eq. (20). One cannot distinguish which source the gluon is radiated from. Since in eq. (14), the first term H_{c1}^1 describes the case that the gluon is emitted from W^+ and the second one H_{c1}^2 describes that the gluon is from W^-. These two terms combined to give probabilities in eq. (20). Note that in this paper, as was done in ref. [4], we also only consider the hard gluon emission. In this case, we find that the interference terms are negligibly small, and we obtain $P_1^1 = P_1^2 \simeq \frac{1}{9}$, $P_1^3 \simeq \frac{5}{18}$ and $P_1^4 \simeq \frac{1}{2}$ from eq. (20). We notice that $|f_1^3>$ or $|f_1^4>$ represents the color singlet formed by two color octets: one is the octet formed by two octets from g_1 and $q_1\overline{q}_4$ which are symmetric in $|f_1^3>$ or antisymmetric in $|f_1^4>$, and the other is from $q_3\overline{q}_2$. In this two states, the four quarks are included in a whole singlet so they do not emerge in two different subsinglets. As we recall that CR means the two original singlets C_1(containing $q_1\overline{q}_2$) and C_2(containing $q_3\overline{q}_4$) are rearranged to make $q_1\overline{q}_4$ and $q_3\overline{q}_2$ enter into two new different subsinglets, our result of CR probability is 2/9. It is interesting to further look at how LUND model treats the hadronization of the states $|f_1^3>$ and $|f_1^4>$. According to the neutral color flow picture of LUND model, such states result in two neutral color flows: one from q_1 to $\overline{q}_2$ via g_1, and the other from q_3 to $\overline{q}_4$, or one from q_1 to $\overline{q}_2$, and the other from q_3 to $\overline{q}_4$ via g_1. These two neutral flows were approximated to singlet string pieces which fragment into hadrons independently. So in LUND model, no difference exists of the hadronization result of $|f_1^3>$ and $|f_1^4>$ from those of the states with no CR. In this sense, the CR probability is also about $\frac{2}{9}$.

C.) For the parton state $q_1\overline{q}_2q_3\overline{q}_4g_1g_2$, to obtain the singlet set that results in the CR, we reduce its color space $3_1\bigotimes 3_2^*\bigotimes 3_3\bigotimes 3_4^*\bigotimes 8_1\bigotimes 8_2$ as follows:

$$\begin{aligned}&(3_1\bigotimes 3_4^*)\bigotimes(3_3\bigotimes 3_2^*)\bigotimes 8_1\bigotimes 8_2\rightarrow(1_{14}\bigoplus 8_{14})\bigotimes(1_{32}\bigoplus 8_{32})\bigotimes 8_1\bigotimes 8_2\\ \rightarrow\ &[1_{14}\bigotimes 1_{32}\bigotimes 8_1\bigotimes 8_2]\bigoplus[1_{14}\bigotimes 8_{32}\bigotimes 8_1\bigotimes 8_2]\bigoplus[8_{14}\bigotimes 1_{32}\bigotimes 8_1\bigotimes 8_2]\\ &\bigoplus[8_{14}\bigotimes 8_{32}\bigotimes 8_1\bigotimes 8_2].\end{aligned}\qquad(21)$$

Obviously the color singlets corresponding to the first three terms in the above equation lead to CR. The last term of eq. (21) can be further reduced as, e.g.

(**a**). $(8_{14}\bigotimes 8_1)\bigotimes(8_{32}\bigotimes 8_2)\rightarrow[1_{8_{14}\bigotimes 8_1}\bigotimes 1_{8_{32}\bigotimes 8_2}]\bigoplus[O_{8_{14}\bigotimes 8_1}\bigotimes O'_{8_{32}\bigotimes 8_2}]$,

(**b**). $(8_{14}\bigotimes 8_2)\bigotimes(8_{32}\bigotimes 8_1)\rightarrow[1_{8_{14}\bigotimes 8_2}\bigotimes 1_{8_{32}\bigotimes 8_1}]\bigoplus[O_{8_{14}\bigotimes 8_2}\bigotimes O'_{8_{32}\bigotimes 8_1}]$,

(**c**). $(8_{14}\bigotimes 8_{32})\bigotimes(8_1\bigotimes 8_2)\rightarrow[1_{8_{14}\bigotimes 8_{32}}\bigotimes 1_{8_1\bigotimes 8_2}]\bigoplus[O_{8_{14}\bigotimes 8_{32}}\bigotimes O'_{8_1\bigotimes 8_2}]$,

(**d**). $8_{14}\bigotimes(8_{32}\bigotimes 8_1\bigotimes 8_2)\rightarrow 1_{8_{14}\bigotimes(8_{32}\bigotimes 8_1\bigotimes 8_2)}$, etc.,

where $O_{8_{14}\bigotimes 8_1}$ ($O'_{8_{32}\bigotimes 8_2}$) denotes the nonsinglet state formed by q_1, $\overline{q}_4$ and g_1 (q_3, $\overline{q}_2$ and g_2), and $O_{8_{14}\bigotimes 8_1}\bigotimes O'_{8_{32}\bigotimes 8_2}$ denotes the color singlet formed by this two nonsinglet states, and so on. The reduction ways (**a**) and (**b**) lead to the same color configurations. The cases (**c**) and (**d**) give a slightly different total CR probability from that of the case **a** or **b**, because all of the singlets reduced in (**c**) and (**d**) are not color rearranged ones, while the first singlets reduced in (**a**) and (**b**) contribute to the total CR probability. As we can see in the following, the difference of total CR probability between this two groups of reduction cases is about 8%. Hence the CR probability slightly depends on the reduction way one chooses. But our current knowledge of QCD is not enough for us to determine which the physical reduction way or the physical singlet set is. As an example, here we discuss the reduction way (**a**). The corresponding color singlet set $\{|f_2^j>,\ j=1,\ \cdots,\ 7\}$ is given by

$$\begin{aligned}&|f_2^1>=\tfrac{1}{3\sqrt{8}}|(\Psi_{1i}\Psi_4^i)(\Psi_{3j}\Psi_2^j)Tr(G_1G_2)>, && |f_2^2>=\tfrac{1}{\sqrt{80}}|(\Psi_{1i}\Psi_4^i)Tr(G_{32}\{G_1,G_2\})>,\\ &|f_2^3>=\tfrac{1}{12}|(\Psi_{1i}\Psi_4^i)Tr(G_{32}[G_1,G_2])>, && |f_2^4>=\tfrac{1}{\sqrt{80}}|(\Psi_{3i}\Psi_2^i)Tr(G_{14}\{G_1,G_2\})>,\\ &|f_2^5>=\tfrac{1}{12}|(\Psi_{3i}\Psi_2^i)Tr(G_{14}[G_1,G_2])>, && |f_2^6>=\tfrac{1}{8}|Tr(G_{14}G_1)Tr(G_{32}G_2)>,\\ &|f^7>=N|(O_{8_{14}\bigotimes 8_1}\bigotimes O'_{8_{32}\bigotimes 8_2})>, &&\end{aligned}\qquad(22)$$

where N is the normalization constant for $|f_2^7>$. The probabilities of $|f_2^i>$ $(i = 1, \cdots, 7)$ are defined by

$$P_2^j = \frac{\int \Phi| < f_2^j|H_{c2}|0 > |^2 d\wp_6}{\sigma_2}, \qquad j = 1, \cdots, 7, \tag{23}$$

where σ_2 is the cross section for the process $e^+e^- \to W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4g_1g_2$, and H_{c2} is given in eq. (15). Note that H_{c2} depends on momentum configurations of partons, so σ_2 and P_2^j are functions of $\sqrt{S}$ and the PQCD parameters. In fig.2, we give the probabilities P_2^j at $\sqrt{S} = 170\ GeV$. One can notice that they are not sensitive to Y_{min}.

From eq. (22), we see that in $|f_2^1>$, $\Psi_{1i}\Psi_4^i$, $\Psi_{3j}\Psi_2^j$ and $Tr(G_1G_2)$ are three color subsinglets within $q_1\overline{q}_4$, $q_3\overline{q}_2$ and g_1g_2 respectively. Obviously this color separated singlet as we call it is not covered by the model of Gustafson and Häkkinen. Its probability is only about 1.3%. In $|f_2^2>$ and $|f_2^3>$, $\Psi_{1i}\Psi_4^i$ is the color singlet within $q_1\overline{q}_4$, while $Tr(G_{32}\{G_1, G_2\})$ and $Tr(G_{32}[G_1, G_2])$ are the singlets formed by three octets: one is symmetric and the other antisymmetric for g_1 and g_2. The situations of $|f_2^4>$ and $|f_2^5>$ are similar to those of $|f_2^2>$ and $|f_2^3>$. In $|f_2^6>$, $Tr(G_{14}G_1)$ $(Tr(G_{32}G_2))$ is the singlet formed by q_1, $\overline{q}_4$ and g_1 (q_3, $\overline{q}_2$ and g_2). According to our definition, at $\sqrt{S} = 170\ GeV$, the total CR probability for $e^+e^- \to W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4g_1g_2$ is about 28% (see fig.2) for the singlet sets (**a**) and (**b**), and about 20% for (**c**) and (**d**). Following a similar discussion as we have made for $e^+e^- \to W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4g_1$, we see that the meaning of the probabilities of these states are different from what Gustafson and Häkkinen imply in their estimation of the CR probabilities. From their analysis, the total probability of color rearrangement for this process is $\sim \frac{3}{9}$ at the partonic level.

Note that Gustafson and Häkkinen regard the CR probability at the partonic level as the upper limit which occurs when the two decay vertices of W^+ and W^- coincide in space-time. The final value should be decreased by taking into account that two W-bosons decay at different space-time points. But in our approach, the CR probabilities at the partonic level already include the effects of non-overlapping decay vertices of two W-bosons. This might be the main difference between these two approaches for the meaning of the CR probability at the partonic level.

V. SUMMARY

The study of CR in hadronic W^+W^- decays is of significance for both precisely measuring the mass of W and clarifying the vacuum structure of QCD. The main goal of this paper is to provide a strict approach to deriving how many CR singlet sets exist in each final parton system and calculating the CR probability at the partonic level. To meet this goal, we use the recursive approach to give the invariant amplitude M_n for the processes $e^+e^- \to W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4 + ng$ $(n = 0,\ 1,\ 2)$ first; Then from M_n, we abstract the corresponding color effective Hamiltonian H_c; Finally, H_c is applied to calculate the CR probability. We find that the CR probabilities are $\frac{1}{9}$, $\frac{2}{9}$ and about 20% $\sim$ 28% (at $\sqrt{S} = 170\ GeV$) for $e^+e^- \to W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4 + ng$ with $n = 0,\ 1$ and 2 respectively.

Summarily, the following points should be noted:

- Our result of the CR probability in PQCD stage is an accurate one at the tree level. It already contains the effect caused by different space-time intervals between the decay vertex of W^+ and that of W^- because our approach is a matrix element method. It shows that the CR probability in PQCD phase is not quite small. Our approach to studying CR seems different from Sjöstrand and Khoze's. The difference may lie in the different definition of CR at the partonic level. We defined CR as the transformation from the original color singlet set to a new one where $q_1\overline{q}_4$ and $q_3\overline{q}_2$ belong to different color subsinglets, while Sjöstrand and Khoze defined that as the "antennae" $\widehat{14}$ and $\widehat{32}$ in momentum space. The "antennae" $\widehat{ij} = \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)}$, where p_i, p_j and k are the momenta of q_i, $\overline{q}_j$ and the gluon radiated from the dipole $q_i\overline{q}_j$.

  In our approach, the color Hamiltonian H_c is abstracted by factorizing the invariant amplitude M into the uniform color part multiplying the momentum one. Then we choose and explore one color singlet set which contains the CR singlets where $q_1\overline{q}_4$ and $q_3\overline{q}_2$ belong to different color subsinglets. By doing projection of CR states to H_c, i.e.,

$< f_{CR}|H_c|0 >$ where f_{CR} denotes the CR singlet, we derive the CR amplitude, then the cross section and the probability. In the procedure of calculating the CR probability, we see that even the color interference terms vanish, there are also CR contributions which are not of interference origin, which shows that the color interference contributes to only part of CR. In Sjöstrand and Khoze's approach, all color indices are summed over in $|M|^2$; the "antenna" terms of $(\widehat{14})$ and $(\widehat{32})$ imply CR only arise in the interference sector of $|M|^2$. The "antennae" can be considered as the sources of further dipole cascading [12], a parallel description of parton cascading. As we remember, the parton shower process can be well described by the GLAP equation where the color indices are summed over at each branching point, so GLAP equation is a probability evolution equation. Another parallel scenario is based on BFKL evolution equation [13] where the color configuration is also smeared. BFKL equation is closely related to the color dipole model. One great advantage of these approaches compared with our matrix element method is that they are easy to be implemented by Monte-Carlo simulation. But in these approaches, the other CR sources besides interference cannot be fully revealed.

We are not sure whether the final event properties predicted from our approach is different from those from Sjöstrand and Khoze's, because in order to produce final hadrons from the partonic states one has to apply a phenomenological model to describe the hadronization process of such color states as $|f_0^2>$, $|f_1^{3,4}>$ and $|f_2^7>$ etc.. Rigorously say, such states are beyond the scope of the currently used fragmentation model.

- Our results for three lowest order cases correspond to three certain reduction ways of the color space which lead to one specific color singlet set for each case that contains the CR states. For $q_1\overline{q}_2q_3\overline{q}_4$ or $q_1\overline{q}_2q_3\overline{q}_4g_1$ case, the corresponding total CR probability that we obtained is the same as what Gustafson and Häkkinen estimated. This is a curious coincidence, because for each of these two cases, only one complete and orthogonal singlet set leads to CR, i.e., all of the CR states belong to the same singlet set and orthogonal to each other. For $q_1\overline{q}_2q_3\overline{q}_4g_1g_2$ case, there are four singlet sets that lead to CR. For two of them the total CR probabilities are the same ($\sim$ 28% at $\sqrt{S} = 170\ GeV$), while for the other two, they are a little smaller ($\sim$ 20% at $\sqrt{S} = 170\ GeV$). The singlet set we choose is one of the former. It implies that in general, for a certain final parton system, there may exist different CR sets and the corresponding CR probabilities. We are far from making the physics choice among them. On the other hand, in this case, some of the CR states are not orthogonal to each other since they do not belong to the same singlet set. For example, one of the rearranged singlets, $|f' >= \frac{1}{8}|Tr(G_{14}G_2)Tr(G_{32}G_1) >$, belongs to a singlet set that is different from the set of $|f_2^6 >$, so they are not orthogonal to each other, as we see $< f'|f_2^6 >= \frac{1}{8} \neq 0$. The probability of a certain CR state is defined in its own set and is meaningless outside it. But Gustafson and Häkkinen do not discriminate different color singlet sets in their estimation where the probabilities of non-orthogonal singlets are added up together and the total CR probability is more than necessary. However, our result displays the same tendency that the gluon emission enlarges the CR probability as predicted by Gustafson and Häkkinen.

- The CR effects observed in the final hadron events still depend on the hadronization mechanism and the vacuum structure, but they originate from the color reconnection at the partonic level. Only when the CR properties at the partonic level are made clear is it possible to investigate the nature of the hadronization mechanism and the vacuum structure.

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Figure Captions

Fig.1: $e^+e^- \to W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4 + ng$ $(n = 0,\ 1,\ \cdots)$ process.

Fig.2: The probabilities P_2^j as a function of Y_{min}.

fig.1

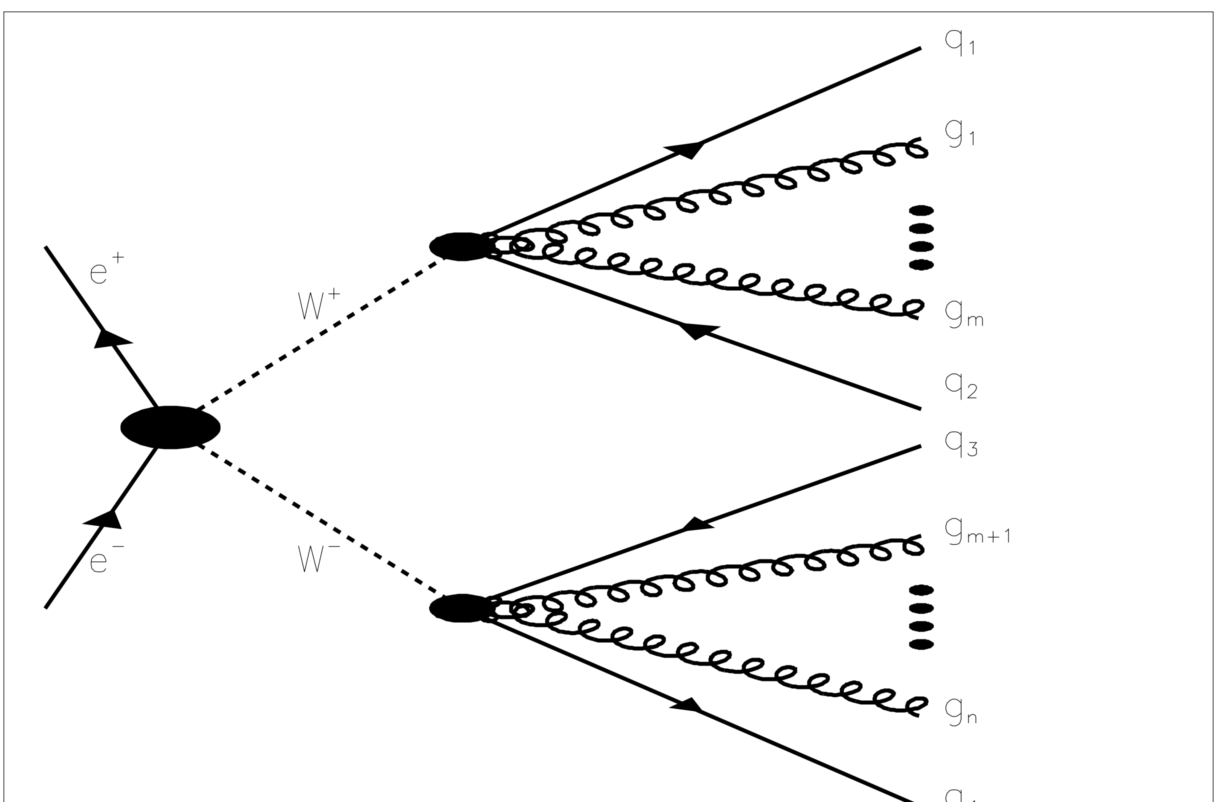
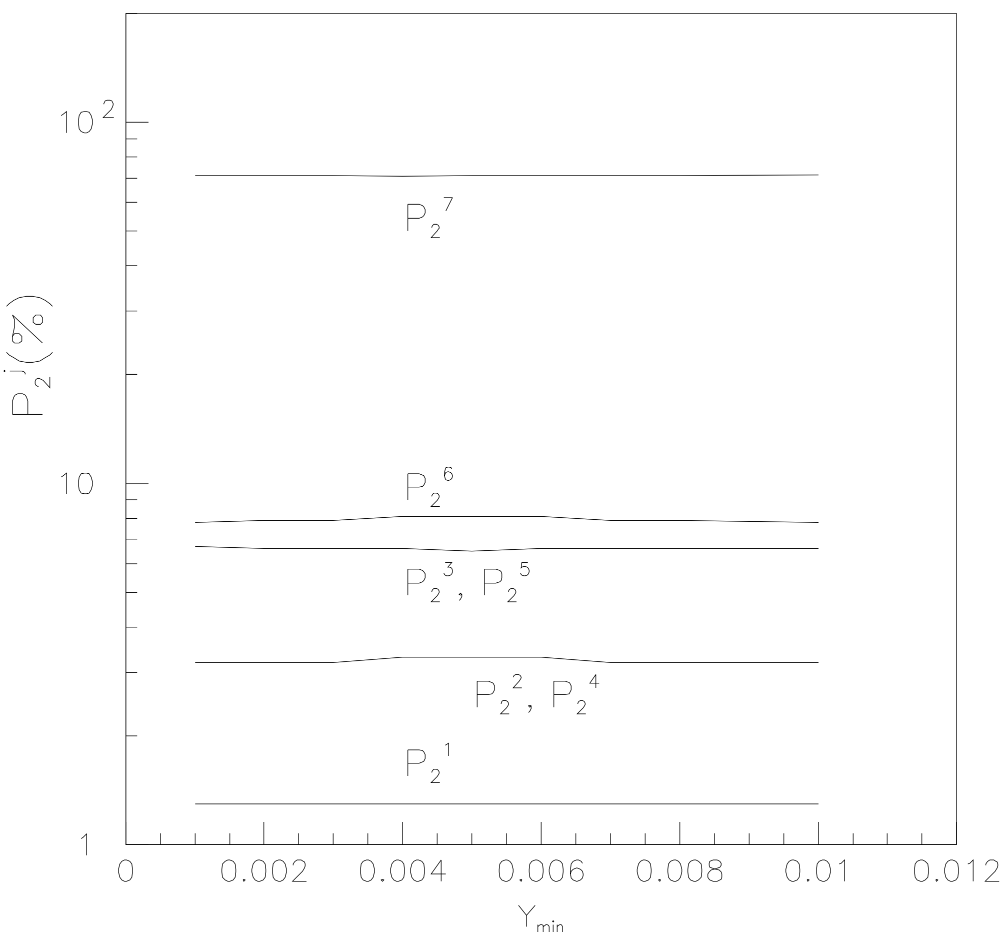


fig.2



10^2
10
1
$P_2^j(\%)$
P_2^7
P_2^6
P_2^3, P_2^5
P_2^2, P_2^4
P_2^1
0
0.002
0.004
0.006
0.008
0.01
0.012
Y_{min}